Problem 5074. Solve in reals:

$$\sqrt{25+9x+30\sqrt{x}}-\sqrt{16+9x+30\sqrt{x-1}}=\frac{3}{x\sqrt{x}}$$

Proposed by Kenneth Korbin, New York, NY

Solution by Ercole Suppa, Teramo, Italy
We must have $x \geq 1$. Observing that $25+9x+30\sqrt{x} = (5+3\sqrt{x})^2$ and $16+9x+30\sqrt{x-1} = \left(5+3\sqrt{x-1}\right)^2$ the given equation is equivalent to

$$\begin{aligned}
&\sqrt{x}-\sqrt{x-1}=\frac{1}{x\sqrt{x}} &&\iff\\
&\frac{1}{\sqrt{x}+\sqrt{x-1}}=\frac{1}{x\sqrt{x}} &&\iff\\
&\sqrt{x}+\sqrt{x-1}=x\sqrt{x} &&\iff\\
&x^2-x=\sqrt{x^2-x} &&\iff\\
&x^2-x=0 \quad \vee \quad x^2-x=1 && (1)
\end{aligned}$$

The solutions of (1) satisfying $x \geq 1$ are $x=1$ and $x=\frac{1+\sqrt{5}}{2}$. □